EMS/DMF Joint Mathematical Weekend
Abstracts

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Contents

Plenary Talks

On the cusp of the new physics: Henri Poincaré and mathematical physics one hundred years ago.

J. J. Gray (Open University)

Henri Poincaré spent much of his working life exploring every branch of mathematical physics, and he wrote about it at every level from the advanced research paper to the popular essay. He was involved at the very start of the move to quantum mechanics, and he famously discovered many of the key ideas in special relativity before Einstein. This paper considers how he tried to shape the mathematical physics at the start of the 20th century.

Approximation properties for groups and C*-algebras

U. Haagerup (University of Copenhagen)

It is classical result in Fourier analysis, that the Fourier series of a continuous function my fail to converge uniformly or even pointwise to the given function. However if one use a summation method as e.g. convergence in Cesaro mean, one actually gets uniform convergence of the Fourier series. This result can easily be generalized first to all abelian (LC = locally compact) groups, and next to all amenable (LC) groups, where in the non-abelian case, the continuous functions on dual group Gm̂ust be replaced by the reduced group C*-algebra of the group G.

In 1994 Jon Kraus and I introduced a new approximation property (AP) for locally compact groups (cf. [2]). The groups having (AP) is the largest class of LC-groups for which a generalized Cesaro mean convergence theorem can hold. The group SL(2,R) has this property, but it was only proven recently by Vincent Lafforgue and Mikael de la Salle, that SL(n,R) and fails to have (AP) for n = 3,4,... In a joint work [1] with Tim de Laat we extend their result by proving that Sp(2,R) and more generally all simple connected Lie groups of real rank ≥ 2 and with finite center do not have the (AP). The proof uses some careful estimates of Jacobi polynomials obtained in collaboration with Henrik Shlichtkrull.

In the talk I will give an introduction to amenabily, weak amenability and the property (AP) for locally compact groups, and the corresponding properties for C*-algebras will also be discussed. Weak amenability is another approximation property for LC-groups, which was introduced in [3].

Key references:

[1] U. Haagerup, T. de Laat, Simple Lie groups without the Approximation Property, arXiv:1201.1250 (2012). To appear in Duke Math. Journal.

[2] U. Haagerup, J. Kraus, Approximation properties for group C*-algebras and group von Neumann algebras. TAMS 344, 667-699 (1994).

[3] M. Cowling and U. Haagerup, Completely bounded multipliers of the Fourier algebra of a simple Lie group of real rank one. Invent. Math. 96, 507-549 (1989).

Propagation in non homogeneous media and applications

H. Berestycki (École des hautes études en sciences sociales, Paris)

The classical theory of the Fisher and Kolmogorov-Petrovsky-Piskunov equation describes spreading properties for a basic reaction-diffusion equation in a homogeneous setting. In particular a well known invasion speed governs the asymptotic speed of propagation. This equation plays an important role in a variety of contexts in ecology, biology and physics.

In this talk I will first recall some of the classical theory for homogeneous F-KPP equations. I will then focus on the effect of inclusion of a line with fast diffusion on biological invasions in the plane. The questions is to know if and how the diffusion on the "road" enhances the propagation in the whole plane. It is found that past a certain threshold for the ratio of diffusivities, the road enhances global diffusion. Several other effects such as transport and reaction on the "road" are discussed. In this talk, I will report on a series of joint works with Jean-Michel Roquejoffre and Luca Rossi.

Probability, and Topology Just to Measure Length

H. Edelsbrunner (IST, Austria)

Decomposition and orientation of graphs

C. Thomassen (Technical University of Denmark)

Latin squares, Steiner triple systems and block designs are structures that can be expressed as graph decompositions. A result of Dehn on rigidity of convex polyhedra motivated an early result on claw decompositions of graphs. In this lecture we focus on the interplay between graph decomposition and graph flow, for example Tutte's flow conjectures. Special emphasis will be on the recent solution of the so-called weak 3-flow conjecture formulated by Jaeger in 1988.

Parallel session I

Diophantine equations, old and new

M. Waldschmidt (Paris VI)

The study of Diophantine equations is among of the oldest topics investigated by mathematicians, it is known that some problems will never be solved, yet fundamental progress has been achieved recently. We survey some of the main results and some of the main conjectures.

Cobordism categories and the A-theory characteristic

G. Raptis (University of Osnabrück)

This talk will be about an interesting link between the cobordism category [3] and Waldhausen's algebraic K-theory of spaces [5]. This link, which was introduced by Bökstedt and Madsen [1], is an infinite loop map connecting the two. The talk will focus on reporting some results [4] relating this map to the A-theory characteristic [2]. The A-theory characteristic of a fibration is a map to algebraic K-theory which can be regarded as a parametrized Euler characteristic of the fibers. Regarding the classifying space of the cobordism category as a moduli space of smooth manifolds stable under extensions by cobordisms, a natural question is whether the A-theory characteristic can be extended to the cobordism category. I will explain why the Bökstedt-Madsen map is such an extension and then I will discuss the analogue of the smooth Riemann-Roch theorem of Dwyer, Weiss and Williams in this context.

This is joint work with W. Steimle [4].

[1] M. Bökstedt and I. Madsen, *The cobordism category and Waldhausen's K-theory*, Preprint, arXiv:1102.4155, 2011.

[2] W. Dwyer, M. Weiss, and B. Williams, *A parametrized index theorem for the algebraic K-theory Euler class*, Acta Math. **190** (2003), no. 1, 1–104.

[3] S. Galatius, U. Tillmann, I. Madsen, and M. Weiss, *The homotopy type of the cobordism category*, Acta Math. **202** (2009), no. 2, 195–239.

[4] G. Raptis and W. Steimle, *On the map of Bökstedt-Madsen from the cobordism category to A-theory*, arxiv:1110.3196, 2011.

[5] F. Waldhausen, *Algebraic K-theory of spaces*, Algebraic and geometric topology (New Brunswick, N.J., 1983), Lecture Notes in Math., vol. 1126, Springer, Berlin, 1985, pp. 318–419.

Epicyclic models and heliocentrism – a commentary on pre-Ptolemaic astronomy

E. Scholz (Wuppertal University)

I want to discuss the question of a possible connection between Aristarchos' heliocentric view and the invention of epicyclic kinematic models for pleantary motion (3rd. century B.C.). Systematic reasons seem to speak in favour of this assumption; but we do lack reliable sources. Although I cannot draw upon new sources and rely completely on the historical work of O. Neugebauer, A. Aaboe, O. Pedersen, G. Toomer and others, I dare to question the common view that heliocentrism at the time between Aristarchos and Hipparchos was nothing but a mathematically empty game of speculations (which was more or less Neugebauer's view). Just to the contrary, the geometrical constructions of epicyclic models were able to provide exactly what one needed for first quantitative phenomenological conclusions from the heliocentric hypothesis. We even have some circumstancial evidence on reasons why later authors (ca. 1st cent. B.C. and later) might have preferred to work with the epicyclic models only, forgetting about the kinematic assumptions leading to them originally. Social historic and philosophical/ideological factors may have added suppression to oblivion.

Loop groups in geometry and singularities

D. Brander (Technical University of Denmark)

Loop group techniques evolving from soliton theory and the study of harmonic maps have for some time now been applied to various problems in geometry: for example to constant mean or Gauss curvature surfaces in space forms, Wilmore surfaces, and higher dimensional isometric immersions of space forms.

Many of these techniques, such as the dressing technique, inverse scattering, and the so-called DPW method depend critically on decompositions of the loop group: such as the Birkhoff factorization, which is an infinite dimensional analogue of the LU decomposition for matrices. If the real form involved is non-compact, then the decomposition is only valid on an open dense subset (the *big cell*) of the relevant component of the loop group. This has often led researchers to describe solutions obtained in this situation as "local solutions", due to the absence of definitive information on the solution at the boundary of the big cell.

In this talk I will try to explain how the solutions can be analyzed at the big cell boundary, and how this analysis can even be used to construct surfaces with prescribed singularities. For example, the stable singularities of timelike and spacelike constant mean curvature surfaces in the 3-dimensional Minkowski space can be classified and constructed through a solution of a *singular Björling problem* [1, 3].

[1] D Brander, *Singularities of spacelike constant mean curvature surfaces in Lorentz-Minkowski space*, Math. Proc. Cambridge Philos. Soc. **150** (2011), 527–556.

[2] D Brander, W Rossman, and N Schmitt, *Holomorphic representation of constant mean curvature surfaces in Minkowski space: Consequences of non-compactness in loop group methods*, Adv. Math. **223** (2010), 949–986.

[3] D Brander and M Svensson, *Timelike constant mean curvature surfaces with singularities*, J. Geom. Anal. (2013), DOI 10.1007/s12220-013-9389-6.

[4] D Brander, *Timelike constant mean curvature surfaces in the anti de Sitter space* $\mathbb{H}^3_1$, (in preparation).

Concavification of free entropy and the additivity problem

P. Biane (CNRS, Institut Gaspard Monge)

We will present a new approach to Voiculescu's free entropy, which allows us in particular to solve the additivity problem.

Parallel session II

Generators of graded rings of modular forms

N. Rustom (University of Copenhagen)

We study graded rings of modular forms over congruence subgroups, with coefficients in a subring A of $\mathbb{C}$, and specifically the highest weight needed to generate these rings as A-algebras. In particular, we determine upper bounds, independent of N, for the highest needed weight that generates the $\mathbb{C}$-algebras of modular forms over $\Gamma(N)$, $\Gamma_1(N)$ and $\Gamma_0(N)$ with some conditions on N. For $N \geq 5$, we prove that the $\mathbb{Z}[1/N]$-algebra of modular forms over $\Gamma_1(N)$ with coefficients in $\mathbb{Z}[1/N]$ is generated in weight at most 3. We give an algorithm that computes the generators, and supply some computations that allow us to state two conjectures concerning the situation over $\Gamma_0(N)$.

Graph-theoretic methods in combinatorial algebraic topology

M. Adamaszek (Univ. Bremen)

Combinatorial topology deals with spaces arising from discrete objects, usually in the form of simplicial, polytopal or other cell complexes. It is therefore not surprising that combinatorial methods are inherent in their study. In this talk I will give two examples of recent results which take this philosophy to the extreme, by reducing topological problems to purely graph-theoretic questions solvable by classical methods of graph theory. Another common theme of both examples is the appearance of flag complexes, that is complexes of cliques in graphs.

The first example concerns the complexity of calculating the homology groups $H_*(K)$ for a simplicial complex K. We exhibit a class of flag complexes with homology generated by spherical classes which can be described by well-understood substructures in graphs. Combinatorial reductions establish NP-hardness of finding such structures, and in consequence NP-hardness of deciding if a simplicial complex K, given by the list of maximal faces, has trivial n-th homology group, for a given index n.

The second example is of a more enumerative kind and contributes to the classification of face vectors of odd-dimensional flag simplicial manifolds. This topic is of current interest due to its connection with the Charney-Davis conjecture and its generalizations. Here the underlying combinatorial techniques go back to some foundational results by Erdös in extremal graph theory.

Impossibility: The Classical Problems

Jesper Lützen (University of Copenhagen)

As pointed out by Hilbert in 1900 some of the most celebrated results in mathematics state that something cannot be done. However, as he also pointed out, this is a recent trend. In former times impossibility results seem to have been considered primarily as meta-statements (that do not call for proofs) concerning the problem solving activity. For example in the case of the three classical problems, on which I shall focus this talk, it took two millennia until Descartes suggested that their impossibility with ruler and compass could be proved. Indeed the new analytic techniques opened up the possibility of such proofs. I shall analyze some of the 17th century attempts to prove the impossibility of the duplication of the cube and the trisection of the angle by ruler and compass (Descartes), and the impossibility of the indefinite and the definite circle quadrature by algebraic means (Wallis, Gregory, Newton and Leibniz). In particular I shall emphasize that these impossibility proofs were an integrated part of a more constructive engagement with the classical problems. As is well known, another two centuries passed until the impossibility of the classical problem was proved by methods that we can still accept. I shall conclude by giving a short account of Wantzel's 1837 proof of the impossibility of the duplication of the cube and the trisection of the angle and its completion by Julius Petersen (1871).

Fibrancy of Symplectic Homology in Cotangent Bundles

T. Kragh (Uppsala University)

In this talk I will discuss symplectic homology of an exact Liouville domain in a cotangent bundle of a closed manifold. I will describe a fiber-wise version and a fibration property. I will then describe how this leads to a Serre type spectral sequence converging to the symplectic homology.

Positive and completely positive maps via free additive powers of probability measures

I. Nechita (CNRS, LPT Toulouse)

We give examples of maps between matrix algebras with different degrees of positivity using ideas from free probability. We discuss applications to entanglement detection in quantum information theory.

Parallel session III

Diophantine approaches of modular curves

P. Parent (I.M.B., Université Bordeaux 1)

Elliptic curves (and modular curves) are the simplest examples of abelian varieties (respectively, Shimura varieties). Yet they are known for having open many doors to deep arithmetic results, the most celebrated instance being probably Wiles' proof of Fermat's Last Theorem. In spite of considerable efforts devoted to their study (and some spectacular results), those objects keep many of their mysteries. In this talk we will try to give an idea of how quite different tools, from modular representations to complex analysis, can be combined to reveal some of their arithmetic features.

Universal operations in higher Hochschild homology

A. Klamt (University of Copenhagen)

We give a definition of formal operations on the higher Hochschild complex of commutative algebras associated to a simplicial set (in the sense of Pirashvili [Pir00]) similar to the construction done by Wahl in [Wah12]. This gives a way to approach the construction of higher string topology operations. We describe the complex of formal operations between the higher Hochschild complexes associated to $X_\bullet$ and $Y_\bullet$. Under some assumptions on $X_\bullet$ and $Y_\bullet$, we identify this complex with the chains on the mapping space of the simplicial sets.

[Wah12] N. Wahl. Universal operations in Hochschild homology. *arXiv preprint arXiv:1212.6498*, 2012.

[Pir00] T. Pirashvili. Hodge decomposition for higher order Hochschild homology. In *Annales Scientifiques de l'Ecole Normale Superieure*, volume 33, pages 151–179. Elsevier, 2000.

The 1874 Controversy between Camille Jordan and Leopold Kronecker

F. Brechenmacher (Université d'Artois & École polytechnique)

During the whole of 1874, Camille Jordan and Leopold Kronecker quarrelled vigorously over the organisation of the theory of bilinear forms. That theory promised a general and homogeneous treatment of numerous questions arising in various 19th-century theoretical contexts, and it hinged on two theorems, stated independently by Jordan and Weierstrass, that would today be considered equivalent. It was, however, the perceived difference between those two theorems that sparked the 1874 controversy. Focusing on this quarrel allows us to explore the algebraic identity of the polynomial practices of the manipulations of forms in use before the advent of structural approaches to linear algebra. The latter approaches identified these practices with methods for the classification of similar matrices. We show that the practices - Jordan's canonical reduction and Kronecker's invariant computation - reflect identities inseparable from the social context of the time. Moreover, these practices reveal not only tacit knowledge, local ways of thinking, but also - in light of a long history tracing back to the work of Lagrange, Laplace, Cauchy, and Hermite - two internal philosophies regarding the significance of generality which are inseparable from two disciplinary ideals opposing algebra and arithmetic. By interrogating the cultural identities of such practices, this study aims at a deeper understanding of the history of linear algebra without focusing on issues related to the origins of theories or structures.

[1] F. Brechenmacher, La controverse de 1874 entre Camille Jordan et Leopold Kronecker, *Revue d'histoire des mathématiques*, **13**, 187-257.

On the mass of asymptotically hyperbolic manifolds

M. Dahl (Kungl Tekniska Högskolan)

As for asymptotically Euclidean manifolds, one can define the mass of an asymptotically hyperbolic manifold. In this talk, I will try to give a background on its definition, positive mass theorems, and the conjectured Penrose inequality. Further, I will address two questions related to this mass. The first one is concerned with the Penrose inequality for hypersurfaces of the hyperbolic space. The second one is the near equality case of the positive mass theorem: if the mass is small in which sense can we say that the metric is close to the hyperbolic metric?

The new results presented come from joint work with Romain Gicquaud and Anna Sakovich.

Analytic subordination for free convolutions

S. T. Belinschi (Queen's University)

The analytic subordination phenomenon for free convolutions was identified by Voiculescu in his first paper on free entropy. Roughly, given two classical probability distributions μ and ν on the real line, Voiculescu's result guarantees the existence of an analytic self-map ω of the complex upper half-plane so that

$$\int_{\mathbb{R}} \frac{1}{z-t}\, d(\mu \boxplus \nu)(t) = \int_{\mathbb{R}} \frac{1}{\omega(z)-t}\, d\mu(t), \quad \Im z > 0. \tag{1}$$

The symbol $\boxplus$ denotes the operation of free additive convolution. This result was later extended by Biane, and further again by Voiculescu, to encompass operator-valued distributions and conditional expectations on tracial W^*-noncommutative probability spaces. In this talk we will identify the subordination functions as limits of iterations of analytic maps on upper half-planes. This will provide on the one hand a method for the (efficient) numerical computation of free convolutions of operator-valued distributions without the need to invert analytic maps (with respect to composition), and on the other it will provide a proof of the subordination result in the absence of a tracial state on the given W^*-noncommutative probability space. We will conclude by showing how this method together with the selfadjoint version (introduced by Greg Anderson) of the Haagerup-Schultz-Thorbjornsen "linearization trick" finds applications in random matrix theory.

Most of the new results presented here are based on joint works [1, 2, 3].

[1] Serban T. Belinschi and Hari Bercovici, *A new approach to subordination results in free probability*, Journal d'Analyse Mathematique 101 (2007), 357–365.

[2] Serban T. Belinschi, Roland Speicher, John Treilhard, and Carlos Vargas, *Operator-valued free multiplicative convolution: analytic subordination theory and applications to random matrix theory*. Preprint 2012.

[3] Serban T. Belinschi, Tobias Mai and Roland Speicher, *Analytic subordination theory of operator-valued free additive convolution and the solution of a general random matrix problem*. Preprint 2013.

Decay of bound states of elliptic PDE's

E. Skibsted (Aarhus University)

Consider a real elliptic polynomial Q on $\mathbb{R}^d$ and the operator $H = Q(-i\nabla) + V$ on $L^2 = L^2(\mathbb{R}^d)$ for a suitable class of real decaying potentials $V = V(x)$. For any bound state $\phi \in L^2$, $(H-E)\phi = 0$, the *critical decay rate* is defined as

$$\sigma_{\text{cri}} = \sup\{\sigma \geq 0 |\, e^{\sigma|x|}\phi \in L^2\}.$$

If $0 < \sigma_{\text{cri}} < \infty$ we prove [HS] the existence of a pair $(\omega, \xi) \in S^{d-1} \times \mathbb{R}^d$ satisfying the following equations with $\sigma = \sigma_{\text{cri}}$:

$$\begin{aligned} &Q(\xi + i\sigma\omega) = E, \\ &\nabla_\xi Q(\xi + i\sigma\omega) = \mu\omega;\ \mu = \omega \cdot \nabla_\xi Q(\xi + i\sigma\omega). \end{aligned} \tag{2}$$

If the equations (2) have a solution for a given $\sigma > 0$ we call σ *exceptional*.

If E is not a critical value of the polynomial Q then indeed $0 < \sigma_{\text{cri}}$, and the bound state ϕ can not be super-exponentially decaying, that is indeed $\sigma_{\text{cri}} < \infty$ (our proof of the latter result requires a somewhat strong decay condition on V).

These results generalize well-known results for one-body Schrödinger operators. Although being very different the exceptional numbers share common features with the possible decay rates for N-body Schrödinger operators, determined by thresholds [FH, IS]. In particular the set of exceptional numbers is in a typical situation countable, in fact finite, and there are no spurious elements, more precisely any exceptional number occurs as a critical decay rate for a bound state with energy E for some potential. For example for the bilaplacian Δ^2, $\sigma = E^{1/4}$ is the only exceptional number for $E > 0$ while $\sigma = (-E/4)^{1/4}$ is the only exceptional number for $E < 0$, and in both cases this number is a critical decay rate for a bound state with energy E for some potential (in fact possibly for a compact support potential).

[FH] R. Froese, I. Herbst, *Exponential bounds and absence of positive eigenvalues for N-body Schrödinger operators*, Commun. Math. Phys. **87** no. 3 (1982/83), 429–447.

[HS] I. Herbst, E. Skibsted, *Decay of bound states of elliptic PDE's*, in preparation.

[IS] K. Ito, E. Skibsted, *Absence of positive eigenvalues for hard-core N-body interactions*, Institut Mittag-Leffler preprint, fall 2012 no. 31.

Parallel session IV

On a conjecture of Lang and Silverman

F. Pazuki (Université Bordeaux 1)

The aim of the talk is to present the state of the art on a conjecture made by S. Lang in [3] about the arithmetic of elliptic curves over number fields and generalized by J. Silverman to higher dimension in [6], claiming the following:

Conjecture (Lang-Silverman): for any number field k and integer $g \geq 1$, there exists a constant $c(k,g) > 0$ such that for any polarized abelian variety (A, D) defined over k of dimension g, for any $k-$rational point P that is non-torsion (and not included in any sub-abelian variety), the Néron-Tate height $\hat{h}_{A,D}(P)$ is greater than $c(k,g)$ times the Faltings height $h_{\mathrm{Falt}}(A/k)$:

$$\hat{h}_{A,D}(P) \geq c(k,g)\, h_{\mathrm{Falt}}(A/k).$$

We will explain the known results of [1, 2, 4, 5] towards this conjecture. We will also give two consequences of this statement regarding bounds on the torsion subgroup of abelian varieties and bounds on the number of k-rational points on curves of genus $g \geq 2$.

[1] David, S., *Minorations de hauteurs sur les variétés abéliennes.* Bull. Soc. Math. France **121** (1993), 509–544.

[2] Hindry, M. and Silverman, J.H., *The canonical height and integral points on elliptic curves.* Invent. Math. **93** (1988), 419–450.

[3] Lang, S., *Elliptic curves: Diophantine analysis.* Grundlehren der Mathematischen Wissenschaften **231** (1978).

[4] Masser, D. W., *Large period matrices and a conjecture of Lang.* Séminaire de Théorie des Nombres, Paris, 1991–92 **116** (1993), 153–177.

[5] Pazuki, F., *Minoration de la hauteur de Néron-Tate sur les variétés abéliennes*, Manusc. Math., to appear.

[6] Silverman, J. H., *Lower bounds for height functions.* Duke Math. J. **51** (1984), 395–403.

Burnside rings and fusion systems

S. P. Reeh (University of Copenhagen)

An extension of the Segal conjecture states that the homotopy classes of stable maps $\{BG_+, BH_+\}$ between classifying spaces of finite groups G and H is isomorphic to a suitable completion $A(G,H)^\wedge$ of the double Burnside module $A(G,H)$ – which is the Grothendieck group of isomorphism classes of finite sets with a right G-action and a free left H-action, with disjoint union as addition, and a composition $A(H,K) \times A(G,H) \to A(G,K)$ corresponding to the composition of maps.

Every saturated fusion system $\mathcal{F}$ over a p-group S has a classifying spectrum $\mathbb{B}\mathcal{F}$, which Kári Ragnarsson has constructed as an infinite mapping telescope of the *characteristic idempotent* $\omega_{\mathcal{F}}$ living in the double Burnside ring $A(S,S)^\wedge \cong \{BS_+, BS_+\}$, and we get a variant of the Segal conjecture for saturated fusion systems, describing the homotopy classes of maps between classifying spectra as $[\mathbb{B}\mathcal{F}_1, \mathbb{B}\mathcal{F}_2] \cong \omega_{\mathcal{F}_2} \circ A(S_1,S_2)^\wedge \circ \omega_{\mathcal{F}_1}$. As time allows, I will also explain my own work on the algebraic properties of these characteristic idempotents, including giving a precise description of their action by multiplication on double Burnside modules.

The two careers of Emmy Noether

C. McLarty (Case Western Reserve University)

Emmy Noether had a notable career as a Nineteenth Century mathematician in quiet Erlangen—while it was already the Twentieth Century according to the calendar. She supervised dissertations, and was elected to the Deutsche Mathematiker Vereinigung and the Circolo Matematico di Palermo, at a time when very few women got such opportunities. In her mid 30s she began another career as a defining mathematician of the Twentieth Century. The talk will explain just what I mean by this distinction between centuries and how it helps to understand Noether's mature style.

A flow approach to special holonomy

H. Weiß (LMU München)

Let M be a compact spin manifold. I discuss a parabolic flow on pairs (g, φ) where g is a Riemannian metric on M and φ a unit g-spinor. This flow is in fact the negative gradient flow of a natural energy functional on the space of such pairs. If $\dim M \geq 3$ then the critical points of this functional are precisely given by those pairs for which φ is g-parallel, in particular g is Ricci-flat and of special holonomy. In two dimensions the critical point structure of the functional is more complicated and the Euler-Lagrange equation is related to further spinorial field equations. If $\dim M = 7$, a pair (g, φ) is equivalent to a so-called positive 3-form on M and hence the flow and the functional may be considered on the space of such positive 3-forms. Critical points now give rise to metrics with holonomy contained in G_2. The G_2-case was in fact the starting point of these investigations and here we obtain a local stability result for the flow near a critical point.

This is based on joint work with Frederik Witt in the G_2-case, resp. with Bernd Ammann and Frederik Witt in the general case.

[1] B. Ammann, H. Weiß, F. Witt, *A spinorial energy functional: critical points and gradient flow*, arXiv:1207.3529.

[2] H. Weiß, F. Witt, *A heat flow for special metrics*, Adv. Math. 231 (2012), no. 6, 3288–3322.

[3] H. Weiß, F. Witt, *Energy functionals and soliton equations for G_2-forms*, Ann. Global Anal. Geom. 42 (2012), no. 4, 585–610.

Free and Boolean Stable Laws

O. Arizmendi (Universtiy of Saarbrucken)

In this talk I will explain some new results regarding free and Boolean stable laws related to infinite divisibility. First I will explain relation of the Marchenko-Pastur law with free and boolean stable laws. Then I will show a reproducing property for free and Boolean stable laws. This generalizes previous results by Biane, for positive free stable laws and by Arizmendi and Perez-Abreu, for symmetric ones. From this properties one can show the free infinite divisibility of Boolean stable laws with index α, for α smaller than 1/2. I will address the general question of free infinite divisibility of Boolean stable laws. I will also show the classical infinite divisibility of positive Boolean stable laws with index α, for α smaller than 1/2. This gives a whole family of examples of classically and freely infinitely divisible probability measures. Finally, as a byproduct I prove a conjecture by Mlotkowksi and Hinz regarding the free Bessel laws. This is a joint work Takahiro Hasebe.

Finite element systems of differential forms

S. H. Christiansen (University of Oslo)

The notion of a finite element system is designed to provide an alternative to Ciarlet's definition of a finite element, adapted to the needs of exterior calculus. It allows for cellular decompositions of space (rather than just simplexes or products thereof) and general functions (rather than just polynomials) yet guarantees compatibility with the exterior derivative and existence of commuting interpolation operators. We review basic definitions and properties [1]. As an application we show how a form of upwinding [2], compatible with the exterior derivative, can be carried out within this framework.

[1] S. H. Christiansen, H. Z. Munthe-Kaas, B. Owren. Topics in structure-preserving discretization. Acta Numerica, No. 20, p. 1-119, 2011.

[2] S. H. Christiansen. Upwinding in finite element systems of differential forms. Foundations of Computational Mathematics, Budapest 2011, London Mathematical Society Lecture Note Series, No. 403, 2012.

Parallel session V

Badly approximable points on manifolds

D. Badziahin (Durham University)

In one dimensional case we have nicely defined set of badly approximable real numbers which can be naturally described in terms of continued fractions. However in higher dimension the picture is not so nice. We have the uncountable family of sets of badly approximable points in R^n for $n \geq 2$. In the last five years our knowledge about the structure of these sets has dramatically improved. In particular, the famous problem of Schmidt was solved. It states that any two (and with some restrictions countably many) such sets have nonempty intersection. In the talk we will concentrate on the last results of Beresnevich, Velani and the speaker about the structure of badly approximable points on non-degenerate manifolds.

Kac-Moody groups via homotopy

J. D. Foley (University of Copenhagen)

Kac-Moody groups generalize Lie groups both in their construction and in their relevance to topics across the mathematical spectrum. Though typically infinite dimensional, they enjoy finiteness properties and their classifying spaces are expressible in terms of a finite number of Lie subgroup classifying spaces. This talk will explore recent developments and current problems for Kac-Moody groups from the perspective of homotopy theory.

Bourbaki's reception of Elie Cartan's work

Renaud Chorlay (SPHERE, CNRS - Uni. Paris Diderot)

Although the published version of the Eléments de Mathématique deals mainly with fundamental structures – with a strong emphasis on algebra – a study of the Bourbaki archives over the period from1934 to 1950 shows how a group of young mathematicians engaged in the collective writing of an up-to-date textbook on mathematical analysis. It can be shown that, in the first period, Elie Cartan's work was meant to play a prominent role in the shaping of this textbook: emphasis on differential forms; local theory of PDEs based on the Cartan-Kähler theory; differentiable manifolds including basic theory of Lie groups and elements of differential geometry. Studying how this project was shaped, and the extent to which it was carried out sheds light on several aspects of the Bourbaki project, and, more generally, on some specific features of mathematical modernity.

Momentum Space for Compact Lie Groups

W. D. Kirwin (Universität zu Köln)

Let K be a compact Lie group. As is well known, $L^2(K)$ can be interpreted as the "position-space" geometric quantization of the cotangent bundle T^*K (which is the gauge-reduced phase space of classical Yang–Mills on a spacetime cylinder). In this talk, I will describe a "momentum-space" quantization of T^*K. I will also explain how this momentum-space quantization is linked to the position-space representation via parallel transport with respect to the Axelrod–Della Pietra–Witten/Hitchin connection in a certain Hilbert bundle. In particular, it is known that parallel transport along a particular geodesic from position space to an intermediate fiber (all of which are Kähler quantizations of T^*K) is the generalized Segal–Bargmann transform. I will explain how this result can be extended to any other interior fiber (thus obtaining generalized generalized Segal–Bargmann transforms), and moreover that when extended to momentum space, parallel transport yields the Peter–Weyl theorem.

Capacity and Error Exponents of Stationary Point Processes under Random Additive Displacements

François Baccelli (INRIA & ENS)

Joint work with Venkat Anantharam

We are given a real-valued discrete-time stationary and ergodic stochastic process, called the noise process. For each dimension n, we get to choose a stationary point process in $\mathbb{R}^n$. Each point is then thought of as giving rise to one daughter point whose displacement from the mother point has the law of a section of length n of the noise process. The displacement vectors are independent and identically distributed over the points. Now consider a decoder, who knows the law of the noise process, the realization of the whole mother point process, and the location of some daughter point. With what probability can the decoder find the mother of this daughter point? The probability of correct decoding is evaluated as a limit over choices of the mother uniformly at random in some exhaustion of $\mathbb{R}^n$. The Shannon regime is the one in which the dimension n tends to infinity while the logarithm of the intensity of the point processes, normalized by dimension, tends to a ! constant.

We first show that this problem exhibits a sharp threshold: if the sum of the asymptotic normalized logarithmic intensity and of the differential entropy rate of the noise process is positive, then the probability of finding the right mother point tends to 0 with n for all mother point processes and all decoding strategies. If this sum is negative, there exist mother point processes, for instance the Poisson process, and decoding strategies, for instance maximum likelihood decoding, for which the probability of finding the right mother tends to 1 with n.

Our concern then is with how quickly the probability of error can be made to go to zero in n. This is a line of attack on the notoriously difficult problem of determining the error exponent function of additive noise channels in information theory. We use large deviations theory to show that in the latter case, if the entropy spectrum of the noise satisfies a large deviations principle, then the error probability goes exponentially fast to 0 with an exponent that we give in closed form in terms of the rate function of the noise entropy spectrum. This is done for two classes of mother point processes: the Poisson process and a Matérn process. These results correspond respectively to the random coding exponent and the expurgated exponent in information theory.

In point process theory this Shannon regime has hardly been considered. Its value becomes apparent from the explicit connection that we also establish between this problem and the study of error exponents in Shannon's additive noise channel with power constraints on the codewords. Each error exponent for the point process problem leads to an exponent for Shannon's additive noise channel through a simple re-parameterization.

More generally, the present paper gives new bounds on error exponents for this and other channels of information theory that hold for all stationary and ergodic noises with the above properties and that match the best known bounds in the white Gaussian noise case in the high signal-to-noise ratio limit.

Derivation of Hartree's theory for generic mean-field Bose gases

M. Lewin (CNRS & Univ. Cergy-Pontoise)

In this talk, I will present a novel strategy to prove the validity of the nonlinear Hartree theory, for the ground state energy of bosonic quantum systems in the limit of a large number N of particles.

More precisely, consider a many-particle Schrödinger operator of the general form

$$H_N = -\Delta + \sum_{j=1}^{N} V(x_j) + \frac{1}{N-1} \sum_{1\leq k<\ell\leq N} w(x_k - x_\ell),$$

acting on $L^2(\mathbb{R}^{dN})$. Then the result says that the infimum $E(N)$ of the spectrum on H_N behaves as

$$\lim_{N\to\infty} \frac{E(N)}{N} = \inf_{\substack{u\in L^2(\mathbb{R}^d)\\ \int_{\mathbb{R}^d}|u|^2=1}} \left\{ \int_{\mathbb{R}^d} \left(|\nabla u(x)|^2 + V(x)|u(x)|^2\right) dx \right.$$
$$\left. + \frac{1}{2}\iint_{\mathbb{R}^d\times\mathbb{R}^d} w(x-y)|u(x)|^2|u(y)|^2\, dx\, dy \right\}.$$

When V tends to infinity at infinity, this can be shown using the strong quantum de Finetti theorem, which gives the structure of infinite hierarchies of k-particles density matrices. Here we deal with the case where some particles are allowed to escape to infinity, leading to a lack of compactness. Our approach is based on two ingredients: (1) a weak version of the quantum de Finetti theorem, and (2) geometric techniques for many-body systems. Our strategy does not rely on any special property of the interaction w between the particles. In particular, our results cover those of Benguria-Lieb and Lieb-Yau for, respectively, bosonic atoms and boson stars.

[1] M. Lewin, P. T. Nam, and N. Rougerie. Derivation of Hartree's theory for generic mean-field Bose gases. *Preprint* arXiv:1303.0981, 2013.

Parallel session VI

Counting rational points on smooth cyclic covers

L. B. Pierce (University of Oxford)

A conjecture of Serre concerns the number of rational points of bounded height on a finite cover of projective space $\mathbb{P}^{n-1}$. This talk will describe a result that verifies Serre's conjecture in the special case of smooth cyclic covers of any degree when the dimension $n \geq 10$, and surpasses the conjecture for such covers of degree at least 3 when $n > 10$. This is achieved by a new upper bound for the number of perfect power values of a polynomial with nonsingular leading form, obtained via a combination of a power sieve and the q-analogue of van der Corput's method.

Always-cartesian cubes and wrong-way maps

R. Eldred (Universität Hamburg)

The goal of this talk is to describe a classification of square diagrams which are homotopy pullbacks (or push outs) and which retain this property when any equivalence-preserving functor is applied. I will explain homotopy pullback/pushout diagrams (and their generalizations to higher dimensions), where this classification problem arose and a conjecture for higher dimensional cubes.

Operations in Functional Analysis

F. Jaëck (CNRS, Laboratoire SPHERE)

Our goal is to understand how mathematicians reinterpreted a diversity of elements they summoned to found the new idea of linear operator at the beginning of the twentieth century. In particular we shall focus on how they re-thought and expressed through multiple modalities the relations between selected elements: nature of the elements involved, rules and relationship will be examined.

Compactifications of reductive groups, non-abelian symplectic cutting and geometric quantisation of non-compact spaces

Johan Martens (QGM, Aarhus University)

We will discuss joint work with Michael Thaddeus (Columbia) on compactifications of reductive groups as moduli spaces of bundles on chains of projective lines, and show how this has a symplectic counterpart in terms of a non-abelian generalisation of Lerman's symplectic cutting operation. We will indicate how to use the latter operation in the context of geometric quantisation of non-compact spaces.

Can Random Sets Help to Assess Precision of Local Algorithms in Image Analysis?

M. Kiderlen (Aarhus University)

We discuss the problem of estimating geometric quantities (such as volume, surface area or Euler Poincaré characteristic) of a suitable subset X in n-dimensional Euclidean space, when only a binary digital image of X is available. We will recall the so-called local algorithms that allow to approximate the desired quantity based on very efficient linear filtering of the image. For finite resolution one cannot expect that these approximations are exact, but one may hope for asymptotic unbiasedness when the resolution tends to infinity.

By applying these algorithms to digitizations of a standard random set model, the stationary Boolean model, we show that many local algorithms are asymptotically biased already for $n = 2$. Using results on large parallel sets in the spirit of Steiner's formula, we even can give approximations of the bias when the resolution is high but finite.

Analysis of a multiscale method for quasilinear elliptic homogenization problems

A. Abdulle (Ecole Polytechnique Fédérale de Lausanne)

In this talk we introduce a numerical homogenization method for a class of nonlinear elliptic problems with multiple scales. Extension of the method relying on interpolation techniques based on the reduced basis methodology will be mentioned [1]. We then discuss the a priori error analysis of the numerical method [2],[3].

[1] A. Abdulle and Y. Bai. Reduced basis finite element heterogeneous multiscale method for high-order discretizations of elliptic homogenization problems, J. Comput. Phys. 231 (2012),7014–7036.

[2] A. Abdulle and G. Vilmart. A priori error estimates for finite element methods with numerical quadrature for nonmonotone nonlinear elliptic problems, Numer. Math. 121 (2012), no. 3, 397–431.

[3] A. Abdulle and G. Vilmart. Analysis of the finite element heterogeneous multiscale method for quasilinear elliptic homogenization problems., to appear in Math. Comp., 2012.

Parallel session VII

'Mixed' problems in Diophantine approximation

S. Harrap (Aarhus University)

It is well known that the rational numbers are dense within the reals. A fundamental aim of the field of Diophantine approximation is to answer the question of 'how dense' they are. A classical theorem of Dirichlet provides a first step; for every irrational x there exists infinitely many rationals p/q for which

$$|x - p/q| \leq 1/q^2. \tag{3}$$

This result is optimal in the sense that one can find a constant c for which the statement does not hold for all irrational x if the left hand side of (3) is replaced by c/q^2. Moreover, Dirichlet's Theorem has the following two dimensional analogue, which is again optimal up to a constant. For every real vector $(x_1, x_2) \in \mathbb{R}^2$ the system of inequalities

$$|x_1 - p_1/q| \leq 1/q^{3/2}, \qquad |x_2 - p_2/q| \leq 1/q^{3/2},$$

is satisfied for infinitely many $p_1, p_2 \in \mathbb{Z}$ and $q \in \mathbb{N}$.

Related to these fundamental results is the Littlewood Conjecture, which states that for every real vector $(x_1, x_2) \in \mathbb{R}^2$ we have

$$\liminf_{q\to\infty} q \cdot \|qx_1\| \cdot \|qx_2\| = 0,$$

where $\| . \|$ denotes the distance to the nearest integer. This conjecture represents one the most profound unsolved problems in Diophantine approximation and has attracted much recent interest.

In their seminal paper [3], de Mathan & Teulié proposed a problem related to the Littlewood Conjecture, realized by retaining the condition that $\|qx_1\|$ is small but replacing the condition on $\|qx_2\|$ with a condition of divisibility of the integer q. This sparked the study of many so-called 'mixed' problems in metric Diophantine approximation.

I will give a brief survey of how the study of these 'mixed' problems has developed, culminating in some recent advances [1, 2], and describe some of the methods commonly used to tackle such questions.

[1] S. Harrap & A. Haynes, *The mixed Littlewood conjecture for pseudo absolute values*, Math. Annal. (to appear). Preprint available at arxiv:1012.0191.

[2] S. Harrap and T. Yusupova: *On a mixed Khintchine problem in Diophantine approximation*, Mosc. J. Comb. Number Theory (to appear). Preprint: arXiv:1201.4694.

[3] B. de Mathan & O. Teulié, *Problèmes Diophantiens simultanés*, Monatsh. Math. **143** (2004), 229–245.

Computational topology for text-mining: application and computations

H. Wagner (Jagiellonian University/IST Austria)

The main topic is at the intersection of computational topology and text-mining. More precisely: we use persistent homology to analyze the structure of similarities within a corpus of text documents. First, some basic concepts from the field of text-mining will be presented. With these tool we map text data into a high dimensional space, which can be treated with topological methods. Then, we give an interpretation of the information captured by persistence. Finally, we overview the computational difficulties and survey some new results, which make the computations feasible even for large datasets.

On the history of mathematics in the 1930s

N. Schappacher (IRMA)

TBA

G. Segal (University of Oxford)

The stochastic Morris-Lecar neuron model and its embedded leaky integrate-and-fire model

S. Ditlevsen (University of Copenhagen)

Stochastic leaky integrate-and-fire models, i.e. one-dimensional mean-reverting diffusions, are popular tools to describe the stochastic fluctuations in the neuronal membrane potential dynamics due to their simplicity and statistical tractability. They have been widely applied to gain understanding of the underlying mechanisms for spike timing in neurons, and have served as building blocks for more elaborate models. Especially the Ornstein-Uhlenbeck process is popular, but also other models like the square-root model or models with a non-linear drift are sometimes applied. However, experimental data show varying time constants, state dependent noise, a graded firing threshold and time-inhomogeneous input, and higher dimensional, more biophysical models are called for.

The stochastic Morris-Lecar neuron is a two-dimensional diffusion which includes ion channel dynamics. We study the dynamics in the model as well as in experimental data from a spinal motoneuron, and relates it to a leaky integrate-and-fire model. The talk is based on joint work with Patrick Jahn, Rune W. Berg, Jørn Hounsgaard and Priscilla Greenwood.

[1] Berg, R. W., Alaburda, A., and Hounsgaard, J. (2007). Balanced inhibition and excitation drive spike activity in spinal halfcenters. *Science*, **315**, 390–393.

[2] Berg, R. W., Ditlevsen, S., and Hounsgaard, J. (2008). Intense synaptic activity enhances temporal resolution in spinal motoneurons. *PLoS ONE*, **3**, e3218.

[3] Ditlevsen, S. and Greenwood, P. (2013). The Morris-Lecar neuron model embeds a leaky integrate-and-fire model. *To appear in Journal of Mathematical Biology.* DOI: 10.1007/s00285-012-0552-7.

[4] Jahn, P., Berg, R. W., Hounsgaard, J., and Ditlevsen, S. (2011). Motoneuron membrane potentials follow a time inhomogeneous jump diffusion process. *Journal of Computational Neuroscience*, **31**, 563–579.

Accelerated boundary integral simulations of particulate and two-phase flows

A.-K. Tornberg (Royal Institute of Technology (KTH), Stockholm)

In micro-fluidic applications where the scales are small and viscous effects dominant, the Stokes equations are often applicable. The suspension dynamics of fluids with immersed rigid particles and fibers are very complex also in this Stokesian regime, and surface tension effects are strongly pronounced at interfaces of immiscible fluids. Simulation methods can be developed based on boundary integral equations, which leads to discretizations of the boundaries of the domain only, and hence fewer unknowns compared to a discretization of the PDE.

Two main difficulties associated with boundary integral discretizations are to construct accurate quadrature methods for singular and nearly singular integrands, as well as to accelerate the solution of the linear systems, that will have dense system matrices. If these issues are properly addressed, boundary integral based simulations can be both highly accurate and very efficient. I will present a spectrally accurate FFT based Ewald method developed for the purpose of accelerating simulations and will discuss its application to simulations of periodic suspensions of rigid particles and rigid fibers in 3D. I will also briefly discuss a method for highly accurate simulations of interacting drops in 2D.

Parallel session VIII

A Distribution Result Related to Automorphic Forms

F. von Essen (University of Copenhagen)

If $\gamma \in SL_2(\mathbb{Z})$ and $|Tr(\gamma)| > 2$, then γ is associated with a geodesic on the surface $\mathbb{H}/SL_2(\mathbb{Z})$. In [1] É. Ghys proves that the linking number between a certain trefoil knot and the geodesic associated with γ (i.e. the number of times the geodesic winds around the trefoil knot) is the Rademacher function $\Phi : SL_2(\mathbb{Z}) \to \mathbb{Z}$. To do this Ghys uses that Dedekind's eta function η, is an automorphic form on $SL_2(\mathbb{Z})$, such that

$$\eta\left(\frac{az+b}{cz+d}\right) = \exp\left(\frac{\pi i}{6}\Phi\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right)(cz+d)^{1/2}\eta(z), \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL_2(\mathbb{Z}).$$

Using Ghys' result P. Sarnak [2] and C. J. Mozzochi showed that the linking number divided by the length of the geodesic is Cauchy distributed.

In the talk there will be a sketch of the proofs of these results, and we will have a look at, how we can use other automorphic forms to get similar results for other groups than $SL_2(\mathbb{Z})$.

[1] Étienne Ghys, Knots and Dynamics. International Congress of Mathematicians. Vol. I, 247-277, Eur. Math. Soc., Zürich, 2007.

[2] Peter Sarnak, Linking Numbers of Modular Knots. Commun. Math. Anal. 8 (2010), no. 2, 136-144.

Homological Reconstruction and Simplification in $\mathbb{R}^3$

U. Bauer (IST Austria)

We consider the problem of deciding whether the persistent homology group of a simplicial pair (K, L) can be realized as the homology $H_*(X)$ of some complex X with $L \subset X \subset K$. We show that this problem is NP-complete even if K is embedded in R^3.

As a consequence, we show that it is NP-hard to simplify level and sublevel sets of scalar functions on S^3 within a given tolerance constraint. This problem has relevance to the visualization of medical images by isosurfaces. We also show an implication to the theory of well groups of scalar functions: not every well group can be realized by some level set, and deciding whether a well group can be realized is NP-hard.

The shaping of a theory of systems of linear inequalities in the 20th century: Contexts, aims and views of mathematics

T. H. Kjeldsen (Roskilde University)

A theory of systems of linear inequalities was developed in bits and pieces primarily during the 20th century. It went on at different times, at several places, for various reasons and under different societal conditions. The development took place in different contexts, spurred by problems in physics, problems in some disciplines of mathematics, the self-understanding of mathematical research and the need of society at large. In the talk we will discuss the development from various perspectives to investigate the significance of the context for shaping the content and the outlook of the theory that was developed. In particular, we will treat contributions by Julius Farkas in analytical mechanics, Herman Minkowski in his book *Geometrie der Zahlen*, American mathematicians with respect to the self-understanding of mathematics in the 1920s in the USA, and the significance of game theory, mathematical programming and the funding of research in the USA in the post WWII period.

Edge state integrals on shaped triangulations

R.M. Kashaev (Geneva University)

A shaped triangulation is a finite triangulation of an oriented (pseudo) 3-manifold where each tetrahedron carries dihedral angles of an ideal hyperbolic tetrahedron. To each shaped triangulation, we associate a quantum partition function in the form of an absolutely convergent state integral which is invariant with respect to shaped $3-2$ Pachner moves and shape gauge transformations generated by total dihedral angles around internal edges through the Neumann–Zagier Poisson bracket. Similarly to Turaev–Viro theory, the state variables live on edges of the triangulation but take their values on the whole real axis. The tetrahedral weight functions enjoy a manifest tetrahedral symmetry. We conjecture that for shaped triangulations of closed 3-manifolds, our partition function is twice the absolute value squared of the partition function of the Teichmüller TQFT defined in [1]. The presentation is based on the work [2].

[1] J. E. Andersen and R. Kashaev, A TQFT from quantum Teichmüller theory, arXiv:1109.6295.

[2] R. Kashaev, F.Luo, G. Vartanov, A TQFT of Turaev–Viro type on shaped triangulations, arXiv:1210.8393.

The stochastics of energy markets

F. E. Benth (University of Oslo)

We analyze various approaches to model the stochastics of forward prices in energy markets. Empirical findings suggest a high degree of idiosyncratic risk between contracts at different maturities, pointing towards infinite-dimensional models for the price dynamics. We introduce a class of infinite-dimensional Levy processes based on subordination, and apply these in an HJM-approach to forward price modeling. Explicit representations of the spot price dynamics are classified. We also investigate a different pricing approach based on Ambit fields. Here, one is modeling the dynamics of forward prices directly rather than as the solution of some stochastic partial differential equation. Applications will be studied, including numerical simulation.

Approximate Electromagnetic Cloaking

M. Vogelius (Rutgers)

A "cloak" is an invisible device that renders a designated area and its (arbitrary) contents invisible to electromagnetic probing as well. I shall describe some of the ideas and techniques behind the "theoretical" construction of electromagnetic (meta-) materials that allow for the creation of such (approximate) cloaks.